Solving Error Problems in Visibility Analysis for Urban Environments by Shifting From a Discrete to a Continuous Approach

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Abstract

Visibility analysis is an important part of the design process of urban environments. Apart from being a key task of every architect and urban designer visibility analysis’ relevance is also strongly confirmed in the disciplines history. Two main mathematical approaches currently exist for visibility analysis, one based on a discrete spatial grid and lines of sight, and one based on continuous space and lines of sight. An estimation of the errors involved in the discrete approach, by using the continuous approach, is a necessary condition for visibility analysis to evolve, as error analysis is in any scientific analysis.

1. Introduction

One of the key tasks for the urban designer and architect is visibility analysis. The relevance of visibility analysis for urban design and planning is confirmed by the work of Cullen [1], Appleyard [2] and Lynch [3]. Studying visual characteristics of the urban environment can be useful to safety, economical aspects, traffic, pedestrian flow and advertising.

‘Measuring’ space is also necessary to secure urban designers a better position in practice. Other players in the building process like traffic planners and property developers often have sharply defined measures and constraints, such as e.g. fixed road patterns and a required number of square meters of parking space respectively. Even the most obvious measures for the urban environment, such as openness & diversity, are rarely quantified in practice.

In literature, quantifications of diversity of the built environment are rare [4, 5], although openness is found in a small number of studies [6]. Furthermore, perceived openness [7] and visual permeability [8] are useful concepts which can be related to locomotion and feelings of safety. Visibility characteristics are also being related to human cognition. In [9], the depth array around a human observer is related to information processing, and affordances are introduced as a link to cognition. There is even a part of the brain (the Parahippocampal Place Area) that is specialized in recognizing enclosure [10]. The PPA responds more strongly to images showing spatial enclosure than to objects. In constrast [11] breaks down objects in geons that the human visual processing system is claimed to process. The isovist as a useful concept for architectural and urban space was first introduced by Benedikt [12]. An isovist simply contains all space that is visible from a certain vantage point. In this paper the steps needed to come to an error estimation of discrete visibility frameworks are presented, by translating to a continuous framework called Isovist-Based Visibility Analysis (IBVA) [13, 14].

2. Relevance of Visual Characteristics

“It is envisaged that with current increases in computational power new algorithms might allow a deeper understanding of urban texture, based on the full exploration of its metric and topological properties. This would contribute to answer the fascinating question …: what is the influence of urban configuration on social life?” [15: p.487].

As a location factor visibility of a shop is only second to accessibility [16]. Apart from their role in human cognitive design, visual characteristics can also play a role in an automated design process using a genetic algorithm, as was established by Fuhrmann and Gotsman [17]. As stated in the introduction, studying visual characteristics of the urban environment can be useful to safety, economical aspects, traffic, pedestrian flow and advertising. Concerning safety, it is known that places that are more visible are in general safer than obscure places. Also shops that are readily accessible and visible from busy areas have an advantage over shops in the less visible outskirts of a
Visibility also plays an important role in traffic safety, as sight is the dominant sense for human mobility.

Putting visibility analysis on the agenda of urban design can lead to identification of relations between apparent unrelated legends and applications. Moreover, it can lead to more coherent and consistent legends and languages for urban design. Only when a sufficient degree of consistency and quantification is achieved, in addition to sufficient empirical support, selected indicators could be promoted to guidelines or even regulations for safeguarding desired visual aspects of the built environment. An estimation of the errors involved is a necessary condition for visibility analysis to evolve, as it is in any scientific analysis.

3. Approaches to Visibility Analysis

From a mathematical and computational point of view, there are two main approaches to visibility analysis. The first approach is based on the concept of a graph consisting of vertices and edges, as depicted in Figure 1. The vertices form a discrete grid in a 2D horizontal plane.

![Figure 1: Two approaches to visibility analysis: graph-based and isovist-based. Adapted from [18]](image)

The edges of the graph represent all possible lines of sight, between all vertices. Within the graph, an isovist consists of all vertices visible to a vantage point (white dots in Figure 1). Most measures only use a single isovist. But there are exceptions such as the clustering coefficient [18] that need multiple isovists. Commonly the analyses are performed on 2D grids in the horizontal plane. The goal is usually to come to some kind of axial analysis, which is useful for predicting pedestrian activity [19].

A continuous approach was developed by [12] who coined the term isovist. The mathematics used allowed for 3D isovists [12], although the calculations were performed in a 2D plane [5]. In [13] a framework is proposed, called Isovist-Based Visibility Analysis (IBVA), based on a slightly different isovist definition. This definition is based on a 3D version of the radial distribution from [12], and can be seen as extending and complementing Benedikt’s framework. Apart from a 3D approach IBVA also allows for error estimations of measures on discrete spatial grids.

In Visibility Graph Analysis (VGA), the line of sight of an isovist is not just the distance $d_{ij}$ from vantage point $i$, but also encompasses the area element $A_j$, resulting in a pair:

$$\left( d_{ij}, A_j \right)$$

In IBVA the unit that is comparable to the pair in Eq. (1), consists of the radial in direction $\phi$ and the infinitesimal angle associated with that radial:

$$\left( R(\phi), d\phi \right)$$

The relation between the two approaches can be expressed mathematically with the aid of Figure 2.

![Figure 2: Connecting discrete VGA to continuous IBVA. Note that $r$ is equivalent to $R(\phi)$ from Eq. (2).](image)

In Figure 2 $\Delta r$ and $\Delta s$ are chosen in such a way that they bound an area equivalent to $A_j$, and also obey Eq. (5) below. To connect the pairs in Eq. (1) and (2) one identifies:

$$A_j = \int dA = \int \int r \, dr \, d\phi \quad \text{and} \quad (3)$$

$$\int d\phi = \Delta \phi = \frac{\Delta s}{d_{ij}} = \frac{\sqrt{A_j}}{d_{ij}} \quad \text{and} \quad (4)$$

since $A_j = \Delta r \cdot \Delta s = \Delta r^2 = \Delta s^2 \quad \text{(5)}$

The equations connecting the pairs, and therefore the two approaches, are:
$R(\phi) = d_0 \pm \frac{1}{2} \Delta r = d_0 \pm \frac{1}{2} \sqrt{A_j}, \quad (6)$

where $\phi = \arccos \left( \frac{\left( \bar{d}_j - \bar{d}_i \right) \cdot \hat{e}_1}{d_{ij}} \right)$, \quad (7)

in which the expression following the $\pm$ sign in Eq. (6) signifies the error in $R(\phi)$, as a result of the approximation of $R(\phi)$ by $d_{ij}$. Thus $\frac{1}{2} \sqrt{A_j}$ is the error made in each radial pointing at $j$, due to the discretization.

### 4 Errors in Angles and Radials

In visibility analyses frameworks based on a discrete spatial grid, such as Visibility Graph Analysis [18], have an angular distribution error, that needs to be addressed. Failing to address this problem introduces errors in the final results. Because space is divided in compartments the angles of the lines of sight around the observer are not equal. See Figure 3.

![Figure 3 Visibility analysis on a discrete grid.](image)

In Figure 3 two pairs of sightlines to the boundary are shown. The angle $\Delta \phi$ between the left pair of sightlines is much smaller than the angle $\Delta \Phi$ between the upper pair of sightlines: $\Delta \phi \neq \Delta \Phi$. Nevertheless the sightlines are treated as equivalent in many measures. In addition an error is introduced in the length of the sightline itself, caused by the discrete spatial grid. Both errors become noticeable when the radial lengths are used in a measure. A measure that recurs frequently in literature is the average distance to the environment [12]. The different errors that occur, are presented in the following by looking at this measure, starting with the error in the distance and radial length.

#### 4.1 Error in the radial length

The discrete version of the average distance is among others found in [4: p.128]:

$$\bar{d}_i = \frac{\sum_{j \in Z_i} d_{ij}}{n_i} , \quad (8)$$

where $j$ runs over each of the $n_i$ vertices (see Figure 2) visible from a vertex $i$. In other words it runs over the entire isovist $Z_i$ or the white plus half the grey squares in Figure 3. This expression is also used in [20: p.25.10]. The error made due to the discrete grid can be calculated with help from Eq. (6), yielding an error:

$$E(\bar{d}_i) = \max \left( \frac{1}{n_i} \sum_{j \in Z_i} \pm \frac{1}{2} \sqrt{A_j} \right) = \frac{1}{2} \sqrt{A_0} \quad (9)$$

where in the last step, it is assumed that all areas $A_j$ are equal to $A_0$ (like in Figure 3). For example, the relative error of the average distance in Figure 3 is approximately 10%, which is substantial. A finer grid improves the error, but is scarcely done, as it comes with a severe penalty on calculation time.

The angular distribution error can not occur here since the angle $\Delta \phi$ does not occur in Eq. 8. However, in a measure that carries the name average, one would expect an angle weighted sum. Therefore it remains unclear what Eq. (8) means, or at least the name average distance is misleading.

#### 4.2 Meaning of average distance

A continuum approach such as IBVA can shed light on what Eq. (8) might mean. First one identifies that the equation (8) in terms of index $j$ can be rewritten in terms of area elements $dA$:

$$\bar{d}_i = \frac{\sum_{j \in Z_i} d_{ij}}{n_i} = \frac{\int \bar{r} \, dA}{\int dA} , \quad (10)$$

where the integration domain $A$ here represents the whole isovist’s area (shaded area in Figure 1). With some calculus Eq. (10) is rewritten in terms of the continuous radial isovist $R(\phi)$, yielding:
\[
\frac{\frac{1}{2} \int_{0}^{2\pi} R_i^3(\phi) \, d\phi}{\int_{0}^{2\pi} R_i^2(\phi) \, d\phi} = \frac{1}{3A} \int_{0}^{2\pi} R_i^3(\phi) \, d\phi. \tag{11}
\]

The denominator in Eq. (11) represents the isovist area \( A \), and the vector \( \vec{x} \) points to the vantagepoint \( i \). For a circular isovist of radius \( R \) this expression (11) evaluates to \( \frac{2}{3}R \), which is again not expected from a measure called the average distance. When looking at the dimensions, Eq. (11) represents a spatial distance [m] as required by Eq. (8). However, what the numerator means, and the quotient as a whole, is still unclear. No simple combination of known measures was found \([13]\) to be equal to Eq. (11). For example the skewness of \( R(\phi) \) can be rewritten to expose the numerator in Eq. (11) \([13]\). Thus if the average distance from Eq. (8) does not have a meaning of its own, for now it seems unlikely that it can be given meaning by expressing it in other measures. Naturally, this does not mean that it can not correlate to other measures and consequently to human perception and experience.

The closest measure (to Eq. 8) in a continuous approach is the average radial length. Note that radials extend to the boundary of the isovist, whereas Eq. (8) considers distances to the entire isovist. Thus it is expected that the mathematical expression of the average radial length \( (AVGR^{2D}) \) will differ from Eq. (11):

\[
AVGR^{2D} = \frac{1}{2\pi} \int_{0}^{2\pi} R_i(\phi) \, d\phi, \tag{12}
\]

which indeed it does. This expression \([13, 5]\) weighs the radials with the angle \( (d\phi) \) they cover. Consequently Eq. (12) does give the values expected from a measure called the average radial length. For example, in case of a circular isovist for which \( R(\phi) = R \), Eq. (12) evaluates to \( R \), as is expected of the average radius of a circle. A discrete version of Eq. (12) looks like:

\[
\overline{d}_i = \frac{\sum_{j \in \partial Z_i} d_{ij} \Delta \phi_{ij}}{\sum_{j \in \partial Z_i} \Delta \phi_{ij}} = \frac{1}{2\pi} \sum_{j \in \partial Z_i} d_{ij} \Delta \phi_{ij} \tag{13}
\]

where the sums traverse over the isovist’s boundary vertices \( \partial Z_i \) (a selection of the grey squares in Figure 3).

4.3 Angular distribution error

Eq. (13) brings us back to the angle issue depicted in Figure 3. For estimating the angular distribution error the difference between the measures of two roomshapes through the same 6 vertices is considered. This will provide an order of magnitude estimation of the error, not a rigorous derivation. The rooms are shown in Figure 4 (Note \( \Delta s = \sqrt{A_0} \)).

![Figure 4 Angular distribution error estimation](image)

The distances to the six visible points are shown in part I of Figure 4. Room II is constructed to have little variation in the angles. Room III is constructed to have large variation in the angles, while still passing through the 6 vertices. The order of magnitude of the error follows from a comparison between the average distance (Eq. 10) and the discretized average radial distance (Eq. 13). The average distance (Eq. 10) is the same for room I, II and III, while the discretized average radial distance (Eq. 13) is varies between room II and II:

\[
\bar{d} \approx 1.138\Delta s \quad \text{for I, II and III}
\]

\[
\bar{d}'_{II} \approx 1.122\Delta s
\]

\[
\bar{d}'_{III} \approx 1.260\Delta s
\]

The relative error, by using Eq. (10) when Eq. (13) is expected is just over 1% in case of room II. However, the relative error is almost 11%, in case of room III, which is substantial.

5. Conclusions

In this paper it is shown that the continuous approach can make measures (Eq. 11 and 12) comparable by translation to the Isovist-Based Visibility Analysis (IBVA) framework. In addition the errors made by using a discrete spatial grid, such as used in Visibility Graph Analysis (VGA), can be calculated explicitly. The order of magnitude of the relative error by neglecting the angular distribution, is found to be 11% for the average distance measure. The relative error made in the radial length depends on the grid size, but was found to be 10% for a typical grid.
These results show the need for a more fundamental continuous approach and open the road to further development of measures. Apart from a needed emphasis on continuous approaches, recent developments [13, 14, 21, 22, 23] also show a shift from 2D to 3D analysis, which open up a whole range of possible new applications.

6. References


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